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A note on variational principles for surface-wave scattering

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Complementary variational formulations are developed for the scattering of a gravity wave by a circular dock. These formulations, which are based on assumed distributions of the radial velocity and the potential, respectively, on the projection of the cylindrical boundary, yield lower and upper bounds to an impedance parameter that determines the difference between the scattered wave for the dock and the corresponding wave for a circular cylinder. Numerical results, using trial functions based on the incident wave, are compared with the results implied by a Galerkin solution (Garrett 1971). The maximum errors in the variational approximations to the total scattering cross-section are found to be of the order of 2 % for a typical depth/radius ratio, draft/depth ratios of 0, $\frac{1}{2}$ and 1, and all wavelengths. The axisymmetric component of the scattering cross-section is found to be very close to the value for scattering by a circular cylinder (dock extending to bottom). The intensity of the scattered wave on the forward axis for long wavelengths and a certain range of the geometric parameters is significantly less than that for a circular cylinder, and may vanish for critical combinations of these parameters.

1. Introduction

We consider the scattering of a gravity wave of amplitude ζ_0 and period $2\pi/\sigma$ by a circular dock of radius a and draft $d-h$ in water of depth d, as in Miles & Gilbert (1968) and Garrett (1971). We refer to these antecedent formulations as I and II, respectively.

The displacement potential and free-surface displacement corresponding to the incident wave,

$$\zeta_i = \zeta_0 \exp(ikr\cos\theta), \tag{1.1}$$

are given by (I (1.2)–(1.6) or II (2.2)–(2.7))

$$\phi(r,\theta,z) = \zeta_0 \sum_{m=0}^{\infty} \epsilon_m i^m \psi_m(r,z)\cos m\theta \quad (\epsilon_m = 2-\delta_{0m}) \tag{1.2}$$

and

$$\zeta(r,\theta) = \zeta_0 \sum_{m=0}^{\infty} \epsilon_m i^m \chi_m(r)\cos m\theta, \quad \chi_m = (\partial\psi_m/\partial z)_{z=d}, \tag{1.3a,b}$$

where ψ_m may be represented in terms of assumed values of either

$$f_m(z) = \partial\psi_m/\partial r \quad (r = a, \quad 0 \leqslant z < h) \tag{1.4}$$

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or $$\mathbf{f}_m(z) = d^{-1}\psi_m \quad (r = a, \quad 0 \leqslant z \leqslant d), \tag{1.5}$$

as in I and II, respectively. These alternative formulations yield complementary variational principles of Schwinger's type, which are especially powerful for the calculation of the scattered wave (cf. Miles 1946, where complementary variational principles for acoustical scattering are developed).

2. First variational principle

The integral equation governing $f_m(z)$, as obtained by separating out the surface-wave mode from II (A1), may be posed in the form,

$$F_m Z_k(z) = \mathscr{F}_{mk}\mathscr{G}_{mk}Z_k(z) + \int_0^h \mathscr{g}_m(z,\zeta)\, f_m(\zeta)\, d\zeta + C_0 \quad (0 \leqslant z \leqslant h), \tag{2.1}$$

where the constraint

$$\int_0^h f_0(z)\, dz = 0 \tag{2.2}$$

must be imposed for $m = 0$, the constant C_0 is related to $\mathbf{f}_0(z)$ by

$$C_0 = \delta_{m0}(d/ah)\int_0^h \mathbf{f}_0(z)\, dz = \delta_{m0}(d/a)\,\mathbf{F}_{00}, \tag{2.3}$$

$$F_m = 2i[\pi k a^2 H_m'(ka)\, Z_k'(d)]^{-1}, \tag{2.4}$$

$$\mathscr{F}_{mk} = d^{-1}\int_0^h f_m(z)\, Z_k(z)\, dz, \tag{2.5}$$

$$\mathscr{g}_m(z,\zeta) = d^{-1}\sum_\alpha \mathscr{G}_{m\alpha} Z_\alpha(z)\, Z_\alpha(\zeta) + h^{-1}\sum_{n=0}^{\infty} \epsilon_n G_{mn}\, c_n(z)\, c_n(\zeta), \tag{2.6}$$

$$c_n(z) = \cos(n\pi z/h) \quad (0 \leqslant z < h) \tag{2.7a}$$

$$= 0 \quad (h < z \leqslant d), \tag{2.7b}$$

$\mathscr{G}_{m\alpha}$, G_{mn}, $Z_k(z)$, and $Z_\alpha(z)$ are given by II (2.29), (2.30), (2.19), and (2.20), respectively, and the α summation is over the positive, real roots of II (2.18) but not the root $\alpha = -ik$. We remark that $\mathscr{G}_{m\alpha}$ and G_{mn} are positive-real, in virtue of which the kernel $\mathscr{g}_m(z,\zeta)$ is positive-definite.

We transform (2.1) to a real integral equation by introducing the scale transformation,

$$\mathscr{f}_m(z) = f_m(z)/\mathscr{F}_{mk}, \tag{2.8}$$

and the real parameter (II (3.5)),

$$\phi_{mk} = (F_m/\mathscr{F}_{mk}) - \mathscr{G}_{mk} \equiv 1/\omega_m; \tag{2.9}$$

then $$\phi_{mk} Z_k(z) = \int_0^h \mathscr{g}_m(z,\zeta)\mathscr{f}_m(\zeta)\, d\zeta + (C_0/\mathscr{F}_{0k}) \quad (0 \leqslant z \leqslant h), \tag{2.10}$$

subject to the additional constraint ($\mathscr{f}_0(z)$ also must satisfy (2.2))

$$d^{-1}\int_0^h \mathscr{f}_m(z) Z_k(z)\, dz = 1. \tag{2.11}$$

We note that $\omega_m = 0$ for the limiting case of the full cylinder ($h = 0$). [We may identify ϕ_{mk} as a pure reactance in series with a complex impedance $\mathscr{G}_{mk}$ in an equivalent circuit in which F_m is (the complex amplitude of) an input voltage, $\mathscr{F}_{mk}$ is a current that is directly proportional to the perturbation potential of the scattered surface wave, and $\frac{1}{2}\phi_{mk}\mathscr{F}^2_{mk}$ is a measure of the stored energy of the internal waves—which are trapped, or non-propagating modes. This analogy may be conceptually valuable, especially in giving the parameter ϕ_{mk} a less abstract character, but it does not appear profitable to pursue it further in the present context.]

Multiplying both sides of (2.10) by $f_m(z)$, integrating over $(0, h)$ and invoking (2.2) and (2.11), we obtain

$$\phi_{mk} = d^{-1}\int_0^h\int_0^h f_m(z)\,g_m(z,\zeta)\,f_m(\zeta)\,d\zeta\,dz \equiv \phi_{mk}\{f_m(z)\}, \tag{2.12}$$

which is stationary with respect to first-order variations of f_m about the solution to the integral equation (2.10), subject to the constraints (2.11) and (2.2) (the constant C_0 is proportional to the Lagrange multiplier that must be introduced in carrying out the standard variational procedure for $m = 0$). Moreover, the approximation $\phi^*_{mk} \equiv \phi_{mk}\{f^*_m\}$ is an upper bound to ϕ_{mk} for any trial function, $f^*_m(z)$, that is in $L^2(0, h)$ and satisfies (2.11) and, for $m = 0$, (2.2). To prove this last assertion, we consider the integral

$$\epsilon \equiv \phi_{mk}\{f^*_m(z) - f_m(z)\} \tag{2.13a}$$

$$= \phi^*_{mk} - 2d^{-1}\int_0^h\int_0^h f^*_m(z)\,g_m(z,\zeta)\,f_m(\zeta)\,d\zeta\,dz + \phi_{mk}, \tag{2.13b}$$

which is non-negative by virtue of the positive-definite character of g_m and vanishes if and only if $f^*_m = f_m$, where f_m satisfies (2.10) and (2.11) and, for $m = 0$, (2.2). Multiplying (2.10) through by $f^*_m(z)$, integrating over $(0, h)$, and substituting the resulting double integral into (2.13b), we obtain

$$\epsilon = \phi^*_{mk} - \phi_{mk} \geqslant 0. \tag{2.14}$$

The foregoing proof is closely related to the question of uniqueness for the solution of (2.10) (the existence and uniqueness of the solution to the original boundary-value problem, as posed by I (1.7)–(1.11), appears to be guaranteed by the investigation of John (1950), but the transformation of this problem to an integral equation of the first kind poses additional difficulties). Suppose that $f_m(z)$ and $f^*_m(z)$ are two distinct solutions of (2.10) and (2.11) and, for $m = 0$, (2.2). Multiplying (2.10) through by $f^*_m(z)$, integrating over $(0, h)$, and invoking the symmetry of $g_m(z, \zeta)$ and the hypotheses that f^*_m satisfies (2.10) and (2.2), we obtain

$$\phi_{mk}\int_0^h f^*_m Z_k\,dz = \int_0^h\int_0^h f^*_m(z)\,g_m(z,\zeta)\,f_m(\zeta)\,d\zeta\,dz \tag{2.15a}$$

$$= \int_0^h f_m(\zeta)\,[\phi^*_{mk} Z_k(\zeta) - (C_0/\mathscr{F}^*_{0k})]\,d\zeta \tag{2.15b}$$

$$= \phi^*_{mk}\int_0^h f_m Z_k\,d\zeta. \tag{2.15c}$$

Invoking (2.11) for both f_m and f_m^*, we obtain $\phi_{mk} = \phi_{mk}^*$. This, in turn, implies, through (2.14), that the positive-definite integral ϵ must vanish, and hence that $f_m^*(z) \equiv f_m(z)$, thereby establishing the uniqueness of the solution.

Finally, we eliminate the constraint (2.11) by substituting f_m from (2.8) into (2.12), invoking (2.5) for $\mathscr{F}_{mk}$, and taking the reciprocal of the result to obtain

$$\omega_m = \frac{\left[\int_0^h f_m(z)\, Z_k(z)\, dz\right]^2}{d\int_0^h\int_0^h f_m(z)\, g_m(z,\zeta) f_m(\zeta)\, d\zeta\, dz} \equiv \omega_m^<\{f_m(z)\}, \tag{2.16}$$

which is equivalent to I (3.2). We emphasize that C_0 drops out of both (2.16) and I (3.2) by virtue of (2.2). The functional $\omega_m^<\{f_m\}$ is stationary with respect to first-order variations of $f_m(z)$ about the true solution to the integral equation (2.1), subject to the constraint (2.2) if $m = 0$, is invariant under a scale transformation of f_m, and yields the lower bound,

$$\omega_m^< \equiv \omega^<\{f_m^*(k)\} \leqslant \omega_m, \tag{2.17}$$

where $\omega_m^< = \omega_m$ if and only if $f_m^*(z) = f_m(z)$, and the inequality is global (rather than local, as in the typical Rayleigh–Ritz approximation) for any $f_m^*(z) \neq f_m(z)$.

3. Second variational principle

Turning now to the formulation of II for $\mathbf{f}_m(z)$, we subtract $(\mathscr{F}_{mk}/\mathscr{G}_{mk})\, Z_k(z)$ from both sides of both II (2.25) and II (2.26) and combine the results to obtain

$$\omega_m \mathscr{F}_{mk} Z_k(z) = \int_0^d \mathbf{g}_m(z,\zeta)\mathbf{f}_m(\zeta)\, d\zeta \quad (0 \leqslant z \leqslant d), \tag{3.1}$$

where

$$\mathscr{F}_{mk} = d^{-1}\int_0^d \mathbf{f}_m(z)\, Z_k(z)\, dz, \tag{3.2}$$

$$\mathbf{g}_m(z,\zeta) = d^{-1}\sum_\alpha \mathscr{G}_{m\alpha}^{-1} Z_\alpha(z)\, Z_\alpha(\zeta) + h^{-1}\sum_{n=0}^{\infty} \epsilon_n G_{mn}^{-} c_n(z)\, c_n(\zeta), \tag{3.3}$$

and the surface-wave mode ($\alpha = -ik$) is omitted from the α-summation. We remark that $\mathbf{g}_m$ is positive-definite and differs from g_m only through the inversion of $\mathscr{G}_{m\alpha}$ and G_{mn}. Proceeding as in § 2, we obtain

$$\omega_m = \frac{d\int_0^d\int_0^d \mathbf{f}_m(z)\, \mathbf{g}_m(z,\zeta)\, \mathbf{f}_m(\zeta)\, d\zeta\, dz}{\left[\int_0^d \mathbf{f}_m(z)\, Z_k(z)\, dz\right]^2} \equiv \omega_m^>\{\mathbf{f}_m(z)\}, \tag{3.4}$$

where $\omega_m^>\{\mathbf{f}_m\}$ is stationary with respect to first-order variations of $\mathbf{f}_m(z)$ about the true solution to the integral equation (3.1), and

$$\omega_m^> \equiv \omega_m^>\{\mathbf{f}_m^*(z)\} \geqslant \omega_m. \tag{3.5}$$

4. Scattered field

The scattered field is given by (I (1.12))

$$\zeta_s \equiv \zeta - \zeta_i \sim \zeta_0 (a/r)^{\frac{1}{2}} \exp(ikr)\, A(\theta) \quad (kr \to \infty), \tag{4.1}$$

where
$$A(\theta) = \sum_{m=0}^{\infty} \epsilon_m A_m \cos m\theta \tag{4.2}$$

is the scattering amplitude. Substituting

$$\lambda_m \equiv \mathscr{F}_{mk}/F_m = 1/(\phi_{mk} + \mathscr{G}_{mk})$$

from (2.9) above into I (4.3), we obtain

$$A_m = -(\tfrac{1}{2}\pi ka)^{-\frac{1}{2}} \exp(-i\pi/4)\,(kaH_m' - \omega_m H_m)^{-1}(kaJ_m' - \omega_m J_m), \tag{4.3}$$

where the argument of J_m and H_m is ka, and J_m' and H_m' are the derivatives with respect to this argument. The total scattering cross-section is given by

$$Q = a\int_0^{2\pi} |A(\theta)|^2 d\theta \equiv a \sum_{m=0}^{\infty} Q_m, \tag{4.4}$$

where
$$Q_m = 2\pi\epsilon_m |A_m|^2 \tag{4.5a}$$

$$= 4\epsilon_m (ka)^{-1} |kaH_m' - \omega_m H_m|^{-2} (kaJ_m' - \omega_m J_m)^2. \tag{4.5b}$$

We also note that
$$Q = -(8\pi a/k)^{\frac{1}{2}} \mathscr{R}\{\exp(i\pi/4)\, A(0)\}. \tag{4.6}$$

Setting $\omega_m = 0$ in (4.3)–(4.5), we recover the known results for the circular cylinder, say $A_m^{(0)}$, $Q^{(0)}$, and $Q_m^{(0)}$. We remark that the zeros of A_m, qua function of ka, lie between the zeros of J_m and J_m' and that $Q_m < 4\epsilon_m/ka$.

The limiting forms of the preceding results as $ka \downarrow 0$ with h/a and d/a fixed are:

$$\omega_0 = O(k^4 h^4), \tag{4.7}$$

$$A(\theta) = (\pi/8)^{\frac{1}{2}} (ka)^{\frac{3}{2}} \exp(i\pi/4)\{-1 + 2(1+\omega_1)^{-1}(1-\omega_1)\cos\theta\}, \tag{4.8a}$$

$$A_m = (2\pi)^{\frac{1}{2}} (ka/2)^{2m-\frac{1}{2}} \{m!(m-1)!\,(m+\omega_m)\}^{-1}(m-\omega_m) \quad (m \geqslant 1), \tag{4.8b}$$

and
$$Q = \tfrac{1}{4}\pi^2 k^3 a^4 \{1 + 2(1+\omega_1)^{-2}(1-\omega_1)^2\}, \tag{4.9}$$

within error factors of $1 + O(k^2a^2 \log ka)$. The axisymmetric component of $A(\theta)$ tends to that of a cylinder by virtue of (4.7), whereas the dipole component may be substantially less than that for a cylinder. We also remark that $A(\theta)$ is a rearward-facing cardioid, and $A(0)$ vanishes like $(ka)^{\frac{7}{2}}$, rather than $(ka)^{\frac{3}{2}}$, if $\omega_1 = \frac{1}{3}$.

A convenient measure of $A(0)$ is the *forward-scattering ratio*,

$$\mathscr{A} \equiv 2\pi a|A(0)|^2/Q = \Big|\sum_{m=0}^{\infty} \epsilon_m A_m\Big|^2 \Big/ \sum_{m=0}^{\infty} \epsilon_m |A_m|^2, \tag{4.10}$$

which reduces to unity for isotropic (axisymmetric) scattering.

5. Numerical tests

The variational formulations of (2.16) and (3.4) permit ω_m, and hence A_m, to be approximated within a determined error. The approximation I (3.5) implies the lower bound,

$$\omega_m^< = \{\sum_\alpha \mathscr{G}_{m\alpha}(\mathscr{F}^*_{m\alpha}/\mathscr{F}^*_{mk})^2 + (h/d) \sum_{n=0}^{\infty} \epsilon_n G_{mn}(F^*_{mn}/\mathscr{F}^*_{mk})^2\}^{-1}, \tag{5.1}$$

where F^*_{mn}, $\mathscr{F}^*_{m\alpha}$, and $\mathscr{F}^*_{mk}$ are given by I (3.6)–(3.8); also $F^*_{mn} \equiv L_{nk}$, as given by II (2.34). The approximation $\mathbf{f}^*_m = Z_k(z)$ implies the upper bound,

$$\omega_m^> = (h/d) \sum_{n=0}^{\infty} \epsilon_n G_{mn}^{-1} L_{nk}^2. \tag{5.2}$$

Letting $ka \downarrow 0$ with d/a and h/d fixed, we obtain

$$\omega_m^< = (h/d)\{m^{-1} + (2/\pi^2)(d/h) \sum_{p=1}^{\infty} p^{-2} G_{mp} \sin^2(p\pi h/d)\}^{-1} \tag{5.3a}$$

$$\doteqdot mh/d - (2m^2/\pi^2)\{m^2 + (\pi a/d)^2\}^{-\frac{1}{2}} \sin^2(\pi h/d), \tag{5.3b}$$

and

$$\omega_m^> = mh/d, \tag{5.4}$$

within error factors of $1 + O(k^2h^2)$. We remark that $\omega_m^<$ and $\omega_m^>$ coincide in both of the limits $h/d \downarrow 0$ and $h/d \uparrow 1$ and exhibit a maximum difference, for fixed d/a, at $h/d = \frac{1}{2}$; this suggests that $h/d = \frac{1}{2}$ provides the most critical test of the variational bounds, at least for $m > 0$ and moderate values of k.

ka	$\omega_0^<$	ω_0	$\omega_0^>$	$\omega_1^<$	ω_1	$\omega_1^>$	$\omega_2^<$	ω_2	$\omega_2^>$
1	$2{\cdot}8 \times 10^{-4}$	$3{\cdot}1 \times 10^{-4}$	$5{\cdot}4 \times 10^{-4}$	0·440	0·443	0·470	0·829	0·839	0·939
2	$3{\cdot}6 \times 10^{-4}$	$4{\cdot}0 \times 10^{-4}$	$6{\cdot}9 \times 10^{-4}$	0·357	0·367	0·393	0·666	0·684	0·779
3	0·012	0·014	0·024	0·254	0·275	0·301	0·466	0·495	0·578
4	0·023	0·026	0·045	0·162	0·194	0·221	0·294	0·331	0·398
5	0·030	0·035	0·059	0·098	0·135	0·163	0·175	0·215	0·267
7	0·029	0·034	0·057	0·035	0·067	0·089	0·061	0·092	0·122
9	0·019	0·022	0·037	0·013	0·033	0·047	0·022	0·041	0·057

TABLE 1. ω_m, as determined by the variational approximations (5.1) and (5.2) and by Garrett's (1971) solution for $h = \frac{1}{2}d$ and $d = \frac{1}{2}a$

Some numerical values of ω_m determined by Garrett (1971) for $h = \frac{1}{2}d$, $d = \frac{1}{2}a$, and $m = 0, 1, 2$ are compared with the corresponding variational approximations in table 1. The corresponding approximations for $m > 2$ are comparable in accuracy with those for $m = 1$ and 2. The larger errors for $m = 0$ have a negligible effect on the axisymmetric scattering amplitude A_0 by virtue of its proximity to $A_0^{(0)}$. The variational approximations to Q_0, Q_1 and Q are compared with the values determined by Garrett's solution in table 2. We use the superscripts $<$ and $>$ to identify the approximations determined by the lower and upper bounds to ω_m, but emphasize that $Q_m^<$ and $Q_m^>$ are not necessarily lower and upper bounds to Q_m (although the difference between $\omega_m^<$ and $\omega_m^>$ may be used to determine the maximum error in either $Q_m^<$ or $Q_m^>$). The maximum errors in $Q^>$ and $Q^<$, as determined by reference to Garrett's solution (for which the error is less than 0·1 %), are 0·3 % and 2 %, respectively. We note the proximity of Q_0 and $Q_0^{(0)}$, which follows from $\omega_0 \ll 1$ and reflects the incompressibility of the fluid under the dock.

Numerical values of ω_1 and Q for the limiting case of a disk ($h = d$) are compared in table 3. The maximum errors in $Q^<$ and $Q^>$ are 2·6 % and 2·1 %, respectively.

ka	$Q_0^<$	Q_0	$Q_0^>$	$Q_0^{(0)}$	$Q_1^<$	Q_1	$Q_1^>$	$Q_1^{(0)}$	$Q^</a$	Q/a	$Q^>/a$	$Q^{(0)}/a$
1	0·9642	0·9643	0·9649	0·9635	0·093	0·091	0·073	0·981	1·064	1·062	1·043	2·000
2	1·9346	1·9347	1·9357	1·9334	0·305	0·314	0·338	0·052	2·341	2·344	2·336	2·718
3	0·6903	0·6896	0·6854	0·6955	1·960	1·976	1·996	1·756	2·850	2·861	2·846	3·033
4	0·0287	0·0289	0·0305	0·0268	1·781	1·771	1·762	1·829	3·030	3·026	2·997	3·213
5	0·6682	0·6687	0·6715	0·6646	0·140	0·134	0·129	0·158	3·207	3·197	3·160	3·330
7		($< 10^{-4}$)			1·138	1·138	1·137	1·139	3·431	3·427	3·403	3·475
9	0·3757	0·3756	0·3751	0·3764	0·175	0·177	0·178	0·174	3·546	3·545	3·535	3·561

TABLE 2. Q_0, Q_1, and Q, as determined by the variational approximations and by Garrett's (1971) solution for $h = \frac{1}{2}d$ and $d = \frac{1}{2}a$, compared with $Q_0^{(0)}$, $Q_1^{(0)}$, and $Q^{(0)}$ for a circular cylinder ($h = 0$)

ka	$\omega^<$	ω_1	$\omega_1^>$	$Q^</a$	Q/a	$Q^>/a$	$Q^{(0)}/a$
0·4	1·00003	1·00012	1·00020	0·12013	0·12015	0·12044	0·415
1	1·0011	1·0044	1·0074	1·0140	1·0156	1·0273	2·000
2	1·015	1·060	1·103	3·008	3·061	3·125	2·718
3	1·064	1·237	1·413	3·800	3·914	3·929	3·033
4	1·15	1·55	1·96	3·885	3·843	3·872	3·213
5	1·28	1·95	2·63	3·961	3·950	4·004	3·330
7	1·59	2·84	4·05	4·077	4·051	4·071	3·475

TABLE 3. Comparison of variational approximations with Garrett's (1971) solution for the limiting case of a disk ($h = d$) with $d = \frac{1}{2}a$

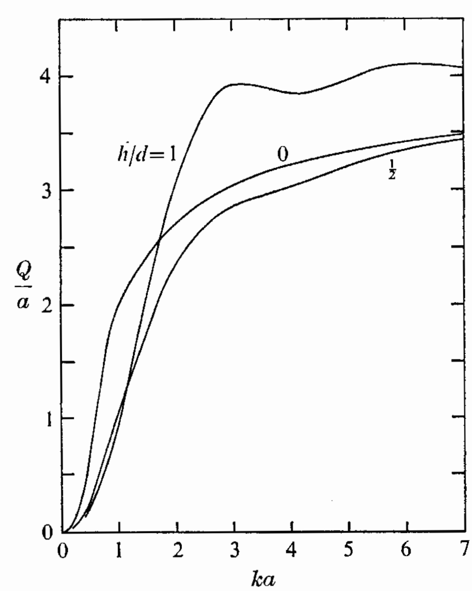

FIGURE 1. The dimensionless scattering cross-section, Q/a, for $d/a = \frac{1}{2}$ (the result for $h/d = 0$ corresponds to a circular cylinder and is independent of d/a).

The variational approximations for this limiting case are significantly superior to those for $h/d = \frac{1}{2}$ if $ka \lesssim 1$, as anticipated above; they are, on the other hand, less accurate for larger k, especially $ka \geqslant 2$, presumably in consequence of the oscillatory character of Q *vs.* ka for $ka \gtrsim 3$. We infer from (4.8), wherein $\omega_m = m$ for $h = d$, that the disk is an isotropic scatterer for sufficiently small ka; see also figure 2, wherein $A \downarrow 1$ as $ka \downarrow 0$.

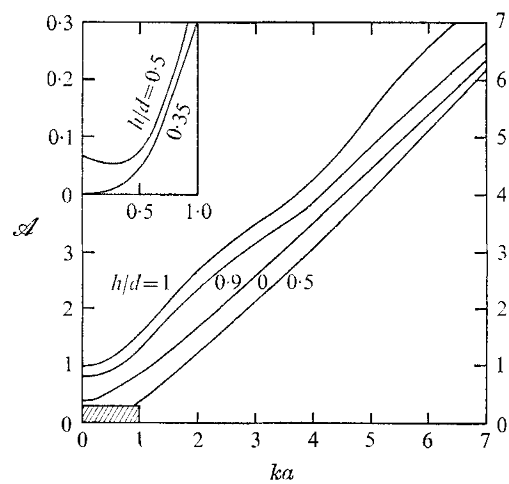

FIGURE 2. The forward-scattering ratio, as defined by (4.10), for $d/a = \frac{1}{2}$. The insert magnifies the cross-hatched area of the main plot.

The results for the opposite limiting case, $h/d \downarrow 0$, are of little interest owing to the fact that our formulation becomes exact in this limit.

The values of Q and $Q^{(0)}$ are plotted in figure 1. The values of Q differ significantly from those presented in I, partially in consequence of an error in I (3.3) (see below) and partially in consequence of a computing error. (J. L. Black, M. C. G. Bray & C. C. Mei, private communication, have also recalculated Q; their results are for different d/a than, but are consistent with, those given here.) The forward-scattering ratio, as defined by (4.10) is plotted in figure 2.

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Corrigenda for

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The term aC_0 should be added to the r.h.s. of (2.7).

The term C_0 should be added to the r.h.s. of both (2.19) and (2.20).

The lower limit for the second summation in (3.3) should be $n = 0$, and the factor 2 should be deleted.

A minus sign should be inserted on the r.h.s. of both (5.6a) and (5.6b), and ζ_0 should be deleted in (5.6a).

The numerical results presented in figures 2–5 are substantially in error. See Black, Mei & Bray (1971) and Garrett (1971).